Mid-Chapter Quiz: Lessons 7-1 through 7- 4

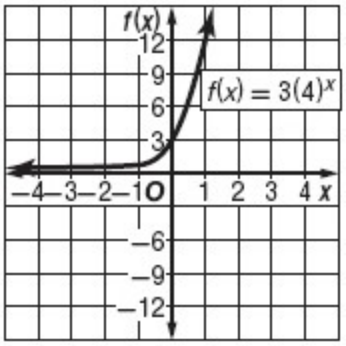
Graph each function. State the domain and range.

1. $f(x) = 3(4)^x$

SOLUTION:

Make a table of values. Then plot the points and sketch the graph.

x	y
–4	0.0117
–2	0.1875
0	3
1	12
2	48



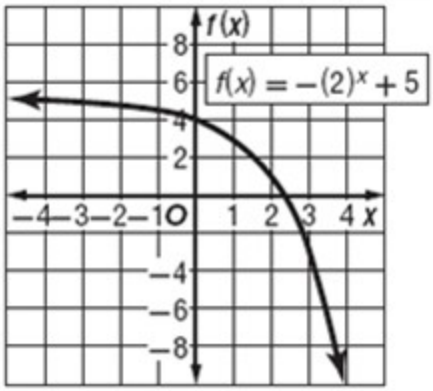
Domain = {all real numbers}; Range = $\{f(x) \mid f(x) > 0\}$

2. $f(x) = -(2)^x + 5$

SOLUTION:

Make a table of values. Then plot the points and sketch the graph.

x	y
–4	4.94
–2	4.75
0	4
1	3
2	1
4	–11



Domain = {all real numbers}; Range = $\{f(x) \mid f(x) < 5\}$

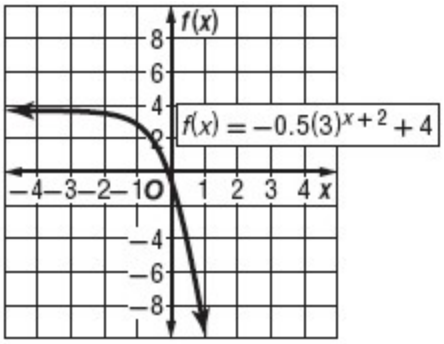
Mid-Chapter Quiz: Lessons 7-1 through 7- 4

3. $f(x) = -0.5(3)^{x+2} + 4$

SOLUTION:

Make a table of values. Then plot the points and sketch the graph.

x	y
–4	3.944
–2	3.5
0	–0.5
1	–9.5



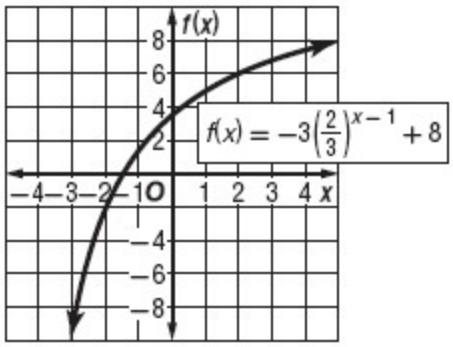
Domain = {all real numbers}; Range = $\{f(x) \mid f(x) < 4\}$

4. $f(x) = -3\left(\frac{2}{3}\right)^{x-1} + 8$

SOLUTION:

Make a table of values. Then plot the points and sketch the graph.

x	y
–3	–7.1875
–1	1.25
0	3.5
2	6
4	7.1111



Domain = {all real numbers}; Range = $\{f(x) \mid f(x) < 8\}$

5. **SCIENCE** You are studying a bacteria population. The population originally started with 6000 bacteria cells. After 2 hours, there were 28,000 bacteria cells.
a. Write an exponential function that could be used to model the number of bacteria after x hours if the number of bacteria changes at the same rate.
b. How many bacteria cells can be expected after 4 hours?

SOLUTION:

a.
Substitute 28000 for y 6000 for a and 2 for t into an exponential function to determine the value of b.

$$\begin{aligned} y &= ab^x \\ 28000 &= 6000b^2 \\ b^2 &= \frac{28000}{6000} \\ b &\approx 2.16025 \end{aligned}$$

The function that models the number of bacteria after x hours is $f(x) = 6000(2.16025)^x$.
b.
Substitute 4 for x in the function $y = 6000(2.16025)^x$.

$$\begin{aligned} y &= 6000(2.16025)^4 \\ y &\approx 130,667 \end{aligned}$$

6. **MULTIPLE CHOICE** Which exponential function has a graph that passes through the points at (0, 125) and (3, 1000)?
A $f(x) = 125(3)^x$
B $f(x) = 1000(3)^x$
C $f(x) = 125(1000)^x$
D $f(x) = 125(2)^x$

SOLUTION:

Find $f(0)$ and $f(3)$ for the function $f(x) = 125(2)^x$.

$$\begin{aligned} f(0) &= 125(2)^0 \\ &= 125 \\ f(3) &= 125(2)^3 \\ &= 125(8) \\ &= 1000 \end{aligned}$$

D is the correct choice.

7. **POPULATION** In 1995, a certain city had a population of 45,000. It increased to 68,000 by 2007.
a. What is an exponential function that could be used to model the population of this city x years after 1995?
b. Use your model to estimate the population in 2020.

SOLUTION:

a.
Substitute 68000 for y 45000 for a and 12 for t into an exponential function to determine the value of b.

$$y = ab^x$$
$$68000 = 45000b^{12}$$
$$b^{12} = \frac{68000}{45000}$$
$$b \approx 1.0350$$

The function that models the population of the city x years after 1995 is $y = 45000(1.0350)^x$.

b.
Substitute 25 for x in the function $y = 45000(1.0350)^x$.

$$y = 45000(1.0350)^{25}$$
$$y \approx 106,346$$

8. **MULTIPLE CHOICE** Find the value of x for $\log_3 (x^2 + 2x) = \log_3 (x + 2)$.
F $x = -2, 1$
G $x = -2$
H $x = 1$
J no solution

SOLUTION:

$$\log_3 (x^2 + 2x) = \log_3 (x + 2)$$
$$x^2 + 2x = x + 2$$
$$x^2 + x - 2 = 0$$
$$(x + 2)(x - 1) = 0$$

$$x + 2 = 0 \quad \text{or} \quad x - 1 = 0$$
$$x = -2 \qquad x = 1$$

Substitute each value into the original equation.

$x = -2$: $\log_3 ((-2)^2 + 2(-2)) \stackrel{?}{=} \log_3 (-2 + 2)$ → $\log_3 0 \stackrel{?}{=} \log_3 0$ ✗

$x = 1$: $\log_3 (1^2 + 2(1)) \stackrel{?}{=} \log_3 (1 + 2)$ → $\log_3 3 = \log_3 3$ ✓

Thus, $x = 1$.
H is the correct choice.

Mid-Chapter Quiz: Lessons 7-1 through 7- 4

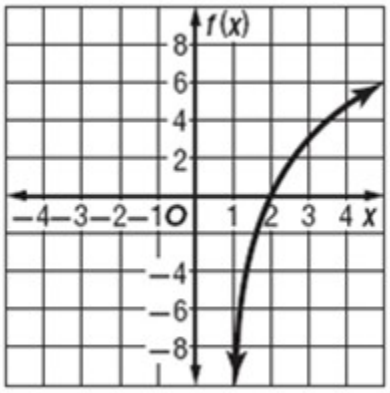
Graph each function.

9. $f(x) = 3 \log_2 (x - 1)$

SOLUTION:

The function is a transformation of the graph of $f(x) = \log_2 x$.

a = 3: The graph is expanded vertically.
h = 1: The graph is translated 1 unit to the right.
k = 0: There is no vertical shift.

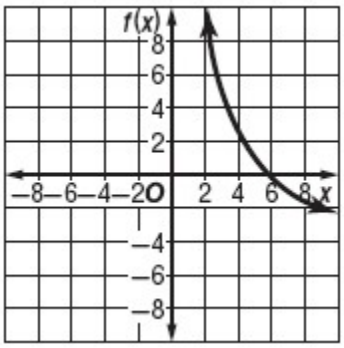


10. $f(x) = -4 \log_3 (x - 2) + 5$

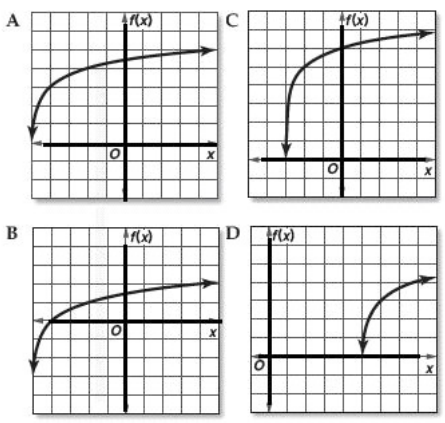
SOLUTION:

The function is a transformation of the graph of $f(x) = \log_3 x$.

a = –4: The graph is reflected across the x-axis.
h = 2: The graph is translated 2 units to the right.
k = 5: The graph is translated 5 units up.



11. **MULTIPLE CHOICE** Which graph below is the graph of the function $f(x) = \log_3 (x + 5) + 3$?



SOLUTION:

The function is a transformation of the graph of $f(x) = \log_3 x$.

h = –5: The graph is translated 5 units to the left.
k = 3: The graph is translated 3 units up.
So, A is the correct choice.

Evaluate each expression.

12. $\log_4 32$

SOLUTION:

$$\log_4 32 = \frac{\log_2 32}{\log_2 4}$$
$$= \frac{\log_2 2^5}{\log_2 2^2}$$
$$= \frac{5}{2}$$

13. $\log_5 5^{12}$

SOLUTION:

$$\log_5 5^{12} = 5$$

14. $\log_{16} 4$

SOLUTION:

$$\log_{16} 4 = \frac{\log_2 4}{\log_2 16}$$
$$= \frac{\log_2 2^2}{\log_2 2^4}$$
$$= \frac{1}{2}$$

15. Write $\log_9 729 = 3$ in exponential form.

SOLUTION:

$$\log_9 729 = 3$$
$$9^3 = 729$$

Solve each equation or inequality. Check your solution.

16. $3^x = 27^2$

SOLUTION:

$$3^x = 27^2$$
$$3^x = 3^6$$
$$x = 6$$

17. $4^{3x-1} = 16^x$

SOLUTION:

$$4^{3x-1} = 16^x$$
$$4^{3x-1} = 4^{2x}$$
$$3x - 1 = 2x$$
$$x = 1$$

18. $\frac{1}{9} = 243^{2x+1}$

SOLUTION:

$$\frac{1}{9} = 243^{2x+1}$$
$$3^{-2} = 3^{10x+5}$$
$$-2 = 10x + 5$$
$$x = -\frac{7}{10}$$

19. $16^{2x+3} < 64$

SOLUTION:

$$16^{2x+3} < 64$$
$$2^{8x+12} < 2^6$$
$$8x + 12 < 6$$
$$8x < -6$$
$$x < -\frac{3}{4}$$

20. $\left(\frac{1}{32}\right)^{x+3} \geq 16^{3x}$

SOLUTION:

$$\left(\frac{1}{32}\right)^{x+3} \geq 16^{3x}$$
$$2^{-5x-15} \geq 2^{12x}$$
$$-5x - 15 \geq 12x$$
$$x \leq -\frac{15}{17}$$

21. $\log_4 x = \frac{3}{2}$

SOLUTION:

$$\log_4 x = \frac{3}{2}$$
$$x = 4^{\frac{3}{2}}$$
$$x = 8$$

Mid-Chapter Quiz: Lessons 7-1 through 7- 4

22. $\log_7 (-x + 3) = \log_7 (6x + 5)$

SOLUTION:

$$\log_7(-x+3) = \log_7(6x+5)$$
$$-x+3 = 6x+5$$
$$x = -\frac{2}{7}$$

Substitute x value into the original equation.

$$x = -\frac{2}{7}$$
$$\log_7\left(-\left(-\frac{2}{7}\right)+3\right) \stackrel{?}{=} \log_7\left(6\left(-\frac{2}{7}\right)+5\right)$$
$$\log_7 \frac{23}{7} = \log_7 \frac{23}{7} \checkmark$$

Thus, $x = -\frac{2}{7}$.

23. $\log_2 x < -3$

SOLUTION:

$$\log_2 x < -3$$
$$x < 2^{-3}$$
$$x < \frac{1}{8}$$

Thus, solution set is $\left\{x \mid 0 < x < \frac{1}{8}\right\}$.

24. $\log_8 (3x + 7) = \log_8 (2x - 5)$

SOLUTION:

$$\log_8(3x+7) = \log_8(2x-5)$$
$$3x+7 = 2x-5$$
$$x = -12$$

Substitute x value into the original equation.

$$x = -12$$
$$\log_8(3(-12)+7) \stackrel{?}{=} \log_8(2(-12)-5)$$
$$\log_8 -29 = \log_8 -29 \times$$

$\log_8 (-29)$ is undefined, so no solution.